1. Neural characterization problem

Goal: characterize the receptive field (RF) using neural responses to white noise or naturalistic stimuli.

Problem: standard estimators are noisy, require lots of data.

Maximum Likelihood Estimator

\[ \hat{k}_{ML} = \mathbf{X}^{-1} \mathbf{Y} \]

simulated example:

\[ \mathbf{X} \sim N(0, \sigma^2 I_\mathbf{X}), \mathbf{Y} = \mathbf{X} \mathbf{W} \]

Gaussian case: zero-mean Gaussian prior + Gaussian likelihood -> evidence is easy to compute!

2. Empirical Bayes (EB)

- Use a prior to regularize RF estimate
- Set hyper-parameters governing that prior by maximum likelihood

Generative model

2-stage estimation procedure

1. Set \( \phi \) by maximizing the “evidence”

\[ \hat{\phi} = \arg \max_{\phi} P(\mathbf{Y} | \phi) P(\phi) \]

2. MAP estimate for \( \mathbf{k} \)

\[ \hat{\mathbf{k}} = \arg \max_{\mathbf{k}} P(\mathbf{Y}, \mathbf{k} | \phi) \]

3. Prior methods (using empirical Bayes)

(A) ridge regression

- Gaussian prior over weights with a common variance
- standard regularization technique: “L2 shrinkage”

(B) automatic relevance determination (ARD)

- Gaussian prior with different variance for each weight
- produces sparse \( \mathbf{k} \)

(C) automatic smoothness determination (ASD)

- Gaussian prior with distance-dependent correlation
- produces smooth \( \mathbf{k} \)

4. Observation

RFs tend to be localized in space-time and spatio-temporal frequency (not just sparse or smooth)

- RF estimates are sparse in both bases
- tend to be smooth

5. Automatic Locality Determination (ALD)

(A) spacetime-localized prior (ALDs)

- diagonal prior in Fourier basis with frequency-dependent variance
- allows large weights only within some space-time region

(B) frequency-localized prior (ALDf)

- diagonal prior with location-dependent variance
- allows large weights only within some region of Fourier space

(C) spacetime & frequency-localized prior (ALDsf)

- “sandwich” together ALDs and ALDf prior covariance matrices

6. Simulations

- 16 min of V1 simple cell data

- Bayesian methods yield significantly lower cross-validation error

- More accurate RF estimates from less data

7. V1 simple cell data

8. Extension: fully Bayesian inference and error bars

- Empirical Bayes fails to take account of uncertainty in hyper-parameters

8.1. Cross-validation error for all methods

8.2. Comparison of average cross-validation error

Conclusions

- novel priors capture localized structure of neural RFs
- automatic setting of hyper-params by empirical Bayes
- more accurate RF estimates from less data

Acknowledgements

We thank Nicole Rust & Tony Movshon for neural data. MP & JWP were supported by the Center for Perceptual Systems.